

Hong Kong Mathematics Olympiad (1997 – 98)

Final Event (Group) – Sample Question

香港數學競賽 (1997 – 98)

決賽項目 (團體) – 樣本題目

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 若 a 是最小的正整數被 3 除時餘 1 而能被 5 整除，求 a 。

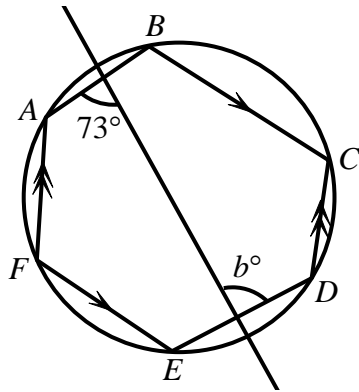
$a =$

If a is the smallest positive integer which gives remainder 1 when divided by 3 and is a multiple of 5, find a .

- (ii) 下圖中， $FA \parallel DC$ 及 $FE \parallel BC$ ，求 b 。

$b =$

In the following diagram, $FA \parallel DC$ and $FE \parallel BC$. Find b .



- (iii) 若 c 是一兩位正整數，其兩位之和是 10 而兩位之積是 25。求 c 。

$c =$

If c is a 2 digit positive integer such that sum of its digits is 10 and product of its digits is 25. Find c .

- (iv) 若 S_1, S_2, \dots, S_{10} 是一由正整數組成的 A.P. 的頭十項使得

$d =$

$S_1 + S_2 + \dots + S_{10} = 55$ 及 $(S_{10} - S_8) + (S_9 - S_7) + \dots + (S_3 - S_1) = d$ 。求 d 。

If S_1, S_2, \dots, S_{10} are the first ten terms of an A.P. consisting of positive integers such that $S_1 + S_2 + \dots + S_{10} = 55$ and

$(S_{10} - S_8) + (S_9 - S_7) + \dots + (S_3 - S_1) = d$. Find d .

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Final Event 1 (Group)

香港數學競賽 (1997 – 98)

決賽項目 1 (團體)

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 若扇形面積 $s = 4 \text{ cm}^2$ ，扇形半徑 $r = 2 \text{ cm}$ 及扇形的弧長 $A = p \text{ cm}$ ，求 p 的數值。

$p =$

If the area of a given sector $s = 4 \text{ cm}^2$, the radius of this sector $r = 2 \text{ cm}$ and the arc length of this sector $A = p \text{ cm}$, find the value of p .

- (ii) 已知 $\frac{a}{2b+c} = \frac{b}{2c+a} = \frac{c}{2a+b}$ 且 $a+b+c \neq 0$ 。若 $q = \frac{2b+c}{a}$ ，求 q 的數值。

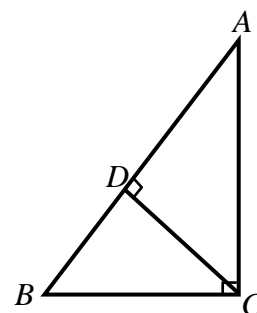
$q =$

Given that $\frac{a}{2b+c} = \frac{b}{2c+a} = \frac{c}{2a+b}$ and $a+b+c \neq 0$. If $q = \frac{2b+c}{a}$, find the value of q .

- (iii) 設直角三角形 ABC 中， CD 是斜邊 AB 上的高， $AC = 3$ ， $DB = \frac{5}{2}$ ， $AD = r$ ，求 r 的數值。

$r =$

Let ABC be a right-angled triangle, CD is the altitude on AB , $AC = 3$, $DB = \frac{5}{2}$, $AD = r$, find the value of r .



- (iv) 若 $x^3 + px^2 + qx + 17 \equiv (x+2)^3 + a$ ，求 a 的數值。

$a =$

If $x^3 + px^2 + qx + 17 \equiv (x+2)^3 + a$, find the value of a .

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Final Event 2 (Group)

香港數學競賽 (1997 – 98)

決賽項目 2 (團體)

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特別聲明，答案須用數字表達，並化至最簡。

(i) 若 $\frac{137}{a} = 0.1\dot{2}3\dot{4}$ ，求 a 的數值。

$a =$

If $\frac{137}{a} = 0.1\dot{2}3\dot{4}$, find the value of a .

(ii) 若 $b = 1999 \times 19981998 - 1998 \times 19991999 + 1$ ，求 b 的數值。

$b =$

If $b = 1999 \times 19981998 - 1998 \times 19991999 + 1$, find the value of b .

(iii) 若參數方程 $\begin{cases} x = \sqrt{3-t^2} \\ y = t-3 \end{cases}$ 可轉換為 $x^2 + y^2 + cx + dy + 6 = 0$ ，

求 c 及 d 的數值。

If the parametric equations $\begin{cases} x = \sqrt{3-t^2} \\ y = t-3 \end{cases}$ can be transformed into

$x^2 + y^2 + cx + dy + 6 = 0$, find the values of c and d .

$c =$

$d =$

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Final Event 3 (Group)

香港數學競賽 (1997 – 98)

決賽項目 3 (團體)

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 在 $\triangle ABC$ 中， $\angle ABC = 2\angle ACB$ ， $BC = 2AB$ 。若 $\angle BAC = a^\circ$ ，求 a 的數值。

In $\triangle ABC$, $\angle ABC = 2\angle ACB$, $BC = 2AB$. If $\angle BAC = a^\circ$, find the value of a .

$a =$

- (ii) 已知 $x + \frac{1}{x} = \sqrt{2}$ ， $\frac{x^2}{x^4 + x^2 + 1} = b$ ，求 b 的數值。

Given that $x + \frac{1}{x} = \sqrt{2}$, $\frac{x^2}{x^4 + x^2 + 1} = b$, find the value of b .

$b =$

- (iii) 若方程 $x + y + 2xy = 141$ 有 c 個正整數解，求 c 的數值。

If the number of positive integral root(s) of the equation $x + y + 2xy = 141$ is c , find the value of c .

$c =$

- (iv) 已知 $x + y + z = 0$ 、 $x^2 + y^2 + z^2 = 1$ 及 $d = 2(x^4 + y^4 + z^4)$ ，求 d 的數值。

Given that $x + y + z = 0$, $x^2 + y^2 + z^2 = 1$ and $d = 2(x^4 + y^4 + z^4)$, find the value of d .

$d =$

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Final Event 4 (Group)

香港數學競賽 (1997 – 98)

決賽項目 4 (團體)

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特別聲明，答案須用數字表達，並化至最簡。

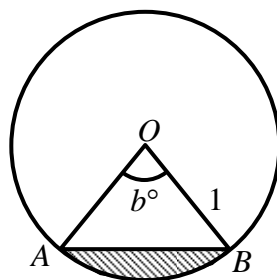
- (i) 若 $0.\dot{1} + 0.0\dot{2} + 0.00\dot{3} + \cdots + 0.00000000\dot{9} = a$ ，求 a 的數值。
(答案以小數表示)。

If $0.\dot{1} + 0.0\dot{2} + 0.00\dot{3} + \cdots + 0.00000000\dot{9} = a$, find the value of a .
(Give your answer in decimal).

$a =$

- (ii) 圖中的圓之圓心為 O ，半徑為 1， A 和 B 是圓形上的點。
已知 $\frac{\text{陰影部分}}{\text{沒有陰影部分}} = \frac{\pi - 2}{3\pi + 2}$ 且 $\angle AOB = b^\circ$ ，求 b 的數值。

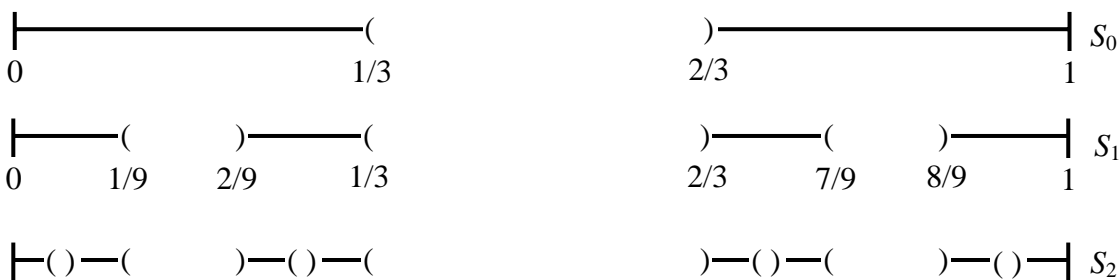
The circle in the figure has centre O and radius 1, A and B are points on the circle. Given that $\frac{\text{Area of shaded part}}{\text{Area of unshaded part}} = \frac{\pi - 2}{3\pi + 2}$ and $\angle AOB = b^\circ$, find the value of b .



$b =$

- (iii) 圖形 S_0, S_1, S_2, \dots 用以下方法構成。把線段 $[0, 1]$ 的中間三分之一取去，得到 S_0 ，把 S_0 的兩條組成線段，每段的中間三分之一取去，得到 S_1 ，把 S_1 的四條組成線段，每段的中間三分之一取去，得到 S_2 。 S_3, S_4, \dots 等用類似方法獲得。求在構成 S_5 的過程中取去的線段的總長度 c 。(答案以分數表示)。

A sequence of figures S_0, S_1, S_2, \dots are constructed as follows. S_0 is obtained by removing the middle third of the interval $[0, 1]$; S_1 by removing the middle third of each of the two intervals in S_0 ; S_2 by removing the middle third of each of the four intervals in S_1 . S_3, S_4, \dots are obtained similarly. Find the total length c of the intervals removed in the construction of S_5 . (Give your answer in fraction).



$c =$

- (iv) 把所有整數用下表的方法編碼。若編碼 101 至 200 的所有整數之和為 d ，求 d 的數值。

All integers are coded as shown in the following table. If the sum of all integers coded from 101 to 200 is d , find the value of d .

整數 Integer	-3	-2	-1	0	1	2	3
編碼 Code	7	5	3	1	2	4	6

$d =$

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Final Event 5 (Group)

香港數學競賽 (1997 – 98)

決賽項目 5 (團體)

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特別聲明，答案須用數字表達，並化至最簡。

- (i) 若 $1 \times 2 \times 3 + 2 \times 3 \times 4 + 3 \times 4 \times 5 + \cdots + 10 \times 11 \times 12 = a$ ，求 a 的數值。

$a =$

If $1 \times 2 \times 3 + 2 \times 3 \times 4 + 3 \times 4 \times 5 + \cdots + 10 \times 11 \times 12 = a$, find the value of a .

- (ii) 已知 $5^x + 5^{-x} = 3$ 。若 $5^{3x} + 5^{-3x} = b$ ，求 b 的數值。

$b =$

Given that $5^x + 5^{-x} = 3$. If $5^{3x} + 5^{-3x} = b$, find the value of b .

- (iii) 已知二次方程 $x^2 + mx + n = 0$ 的兩根為 98 和 99，且 $y = x^2 + mx + n$ 。若 x 取 0、1、2、...、100，則有 c 個 y 的數值能被 6 整除。求 c 的數值。

$c =$

Given that the roots of quadratic equation $x^2 + mx + n = 0$ are 98 and 99 and $y = x^2 + mx + n$. If x takes on the values of 0, 1, 2, ..., 100, then there are c values of y that can be divisible by 6. Find the value of c .

- (iv) 在圖中， $ABCD$ 為一正方形， $BF \parallel AC$ ，且 $AEFC$ 為一菱形。若 $\angle EAC = d^\circ$ ，求 d 的數值。

$d =$

In the figure, $ABCD$ is a square, $BF \parallel AC$, and $AEFC$ is a rhombus. If $\angle EAC = d^\circ$, find the value of d .

